Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers

Student Outcomes

- Students use properties of operations to add and subtract rational numbers without the use of a calculator.
- Students recognize that any problem involving addition and subtraction of rational numbers can be written as a problem using addition and subtraction of positive numbers only.
- Students use the commutative and associative properties of addition to rewrite numerical expressions in different forms. They know that the opposite of a sum is the sum of the opposites (e.g., $-(3 + (-4)) = -3 + 4$).

Lesson Notes

This lesson is the first of a two-day lesson using the properties of operations to add and subtract rational numbers. The lesson begins with a focus on representing the opposite of a sum as the sum of its opposites so that students may more efficiently arrive at sums and differences of rational numbers. The focus includes a representation of negative mixed numbers so that students conceptualize a negative mixed number as a negative integer plus a negative fraction. Students often mistakenly add a negative mixed number to a positive whole number by adding the negative whole number part of the mixed number to the positive whole number but then erroneously representing the fractional part of the negative mixed number as a positive number.

The following is an example of the properties and how they are used in this lesson.

$$-13\frac{5}{7} + 6 - \frac{2}{7}$$

$$= -13\frac{5}{7} + 6 + \left(-\frac{2}{7}\right) \qquad \textit{Subtracting a number is the same as adding its inverse.}$$

$$= -13 + \left(-\frac{5}{7}\right) + 6 + \left(-\frac{2}{7}\right) \qquad \textit{The opposite of a sum is the sum of its opposite.}$$

$$= -13 + \left(-\frac{5}{7}\right) + \left(-\frac{2}{7}\right) + 6 \qquad \textit{Commutative property of addition}$$

$$= -13 + (-1) + 6 \qquad \textit{Associative property of addition}$$

$$= -14 + 6$$

$$= -8$$

Recall the Properties of Operations from Table 3 of the Common Core State Standards.

Table 3. The Properties of Operations. Here a, b, and c stand for arbitrary numbers.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Commutative property of multiplication	$a \cdot b = b \cdot a$
Multiplicative identity property of 1	$a \cdot 1 = 1 \cdot a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$
Distributive property of multiplication over addition	$a \cdot (b + c) = a \cdot b + a \cdot c$

Classwork

Fluency Exercise (6 minutes): Integer Addition

Photocopy the attached 2-page fluency-building exercises so that each student receives a copy. Before students begin, inform them that they may not skip over questions and that they must move in order. Time students, allowing one minute to complete Side A. After one minute, discuss the answers. Before administering Side B, elicit strategies from those students who were able to accurately complete many problems on Side A. Administer Side B in the same fashion, and review the answers. Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint.

Opening Exercise (3 minutes): Recall of a Number's Opposite

This warm-up prepares students for Exercise 1. Ahead of time, post a large number line on the side wall (either in poster form or with painter's tape).

Scaffolding:

Select specific cards to give to students to challenge them at their level.

As students enter the room, hand them a small sticky note with a rational number on it. Ask them to "find their opposite." (Sticky notes are such that each signed number has a "match" for opposite.) Students pair up according to opposites, walk to the number line on the side wall, and stick their numbers in the correct locations on the number line. The class comes to a consensus that all numbers are placed in the correct location.

Example 1 (4 minutes): The Opposite of a Sum is the Sum of its Opposites

Have the following statement up on the board: *"The opposite of a sum is the sum of its opposites."* Tell students we are going to use some numbers from the Opening Exercise to investigate this statement.

Ask two pairs of students (who were partners from the Opening Exercise) to come to the front of the room. (Choose students who had rational numbers that were integers, as they will be easier to understand in this example.) Have one person from each pair write their numbers on the board; let's say they were 7 and -2. Find the sum, $7 + -2 = 5$, and then find the opposite of the sum, -5. Now have partners write their numbers on the board, -7 and 2, and then find the sum of these opposites, -5. Now it is clear that the opposite of the sum is equal to the sum of the opposites.

Example 1: The Opposite of a Sum is the Sum of its Opposites

Explain the meaning of "The opposite of a sum is the sum of its opposites." Use a specific math example.

Rational Number	Rational Number	Sum	Opposite of the Sum
7	−2	5	−5

Opposite Rational Number	Opposite Rational Number	Sum
−7	2	−5

It means that if you have a sum and want to take the opposite, for instance, $-(7 + (-2))$, you can rewrite it as the sum of each addend's opposite, $-7 + 2$.

> *Scaffolding:*
>
> Display an anchor poster in the classroom to show the meaning of "the opposite of a sum is the sum of its opposites." Label the "opposite" and "sum" in a specific math example.

Exercise 1 (5 minutes)

Have students arrive at an answer to the following. Students share their different strategies with the class. The class members discuss the strategies they used. They determine which are most efficient, which ways are less likely to cause errors and confusion, and whether they were able to reach the correct answer. If no students share the solution method on the right, share it with the class.

Exercise 1

Represent the following expression with a single rational number.

$$-2\frac{2}{5} + 3\frac{1}{4} - \frac{3}{5}$$

Two Possible Methods:

$$-2\frac{8}{20} + 3\frac{5}{20} - \frac{12}{20}$$

$$-\frac{48}{20} + \frac{65}{20} - \frac{12}{20}$$

$$\frac{17}{20} - \frac{12}{20}$$

$$\frac{5}{20} \ or \ \frac{1}{4}$$

OR

$$-2\frac{2}{5} + 3\frac{1}{4} + \left(-\frac{3}{5}\right)$$

$$-2\frac{2}{5} + \left(-\frac{3}{5}\right) + 3\frac{1}{4} \qquad commutative\ property$$

$$-2\frac{5}{5} + 3\frac{1}{4}$$

$$-3 + 3\frac{1}{4}$$

$$\frac{1}{4}$$

After students share their strategies, the following are questions that may guide the whole-group discussion. Student responses to the suggested discussion questions will vary.

- ▪ Was it difficult for you to add the mixed numbers with different signs and denominators? Why or why not?
- ▪ Were you able to arrive at the correct answer?
- ▪ Which method do you prefer?
- ▪ Which method is more challenging for you?

<div style="background:#d6e4f0">Example 2 (5 minutes): A Mixed Number Is a Sum</div>

The following example allows students to focus on a mixed number as a sum. Looking at $2\frac{2}{5}$, they think about how it can be rewritten using addition. (It means $2 + \frac{2}{5}$.) Once students represent it as a sum, they recognize that $-2\frac{2}{5}$ means $-2 + \left(-\frac{2}{5}\right)$. The following is a possible lead-in question.

- ▪ $-2\frac{2}{5}$ is the opposite of $2\frac{2}{5}$. How can we show "the opposite of a sum is the sum of its opposites" with the number $-2\frac{2}{5}$? How do we model it on a number line?

Example 2: A Mixed Number Is a Sum

Use the number line model shown below to explain and write the opposite of $2\frac{2}{5}$ as a sum of two rational numbers.
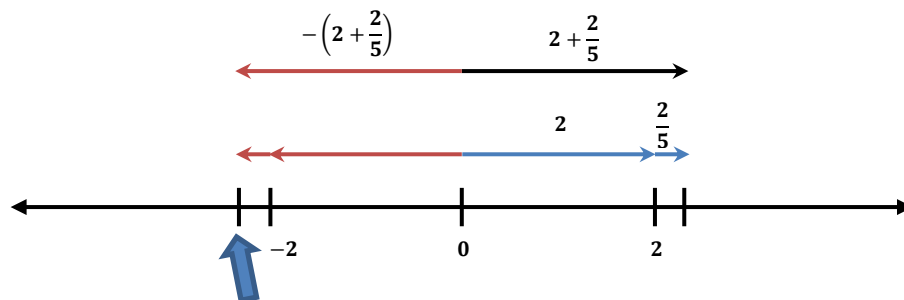
The opposite of a sum (top single arrow pointing left) and the sum of the opposites correspond to the same point on the number line.

The opposite of $2\frac{2}{5}$ is $-2\frac{2}{5}$.

$-2\frac{2}{5}$ written as the sum of two rational numbers is $-2 + \left(-\frac{2}{5}\right)$.

Exercise 2 (2 minutes)

Students independently rewrite each mixed number as the sum of two signed numbers. The teacher circulates the room, providing assistance as needed. After two minutes, discuss the answers as a whole group.

> *Scaffolding:*
>
> Provide students with a laminated copy of the number line model used in Example 2. Also provide number lines so they can represent each of the following as a sum on the number line.

Exercise 2

Rewrite each mixed number as the sum of two signed numbers.

a. $-9\frac{5}{8}$ b. $-2\frac{1}{2}$ c. $8\frac{11}{12}$

$-9 + \left(-\frac{5}{8}\right)$ $-2 + \left(-\frac{1}{2}\right)$ $8 + \frac{11}{12}$

Exercise 3 (2 minutes)

MP.8

Students independently use the reverse process to represent each sum or difference as a mixed number. The teacher circulates the room providing assistance as needed. After two minutes, discuss the answers as a whole group.

Exercise 3

Represent each sum as a mixed number.

a. $-1 + \left(-\frac{5}{12}\right)$ b. $30 + \frac{1}{8}$ c. $-17 + \left(-\frac{1}{9}\right)$

$-1\frac{5}{12}$ $30\frac{1}{8}$ $-17\frac{1}{9}$

Note: Exercises 3 and 4 are designed to provide students with an opportunity to practice writing mixed numbers as sums so they can do so as the need arises in more complicated problems.

Exercise 4 (5 minutes)

Students work independently to solve the problem below. Then student volunteers share their steps and solutions with the class. Note, the solution below includes just one possible solution method. However, a common mistake is for students to arrive at an *incorrect answer of* $-5\frac{1}{8}$. As needed, revisit subtracting a mixed number from a whole number.

Exercise 4

Mr. Mitchell lost 10 pounds over the summer by jogging each week. By winter, he had gained $5\frac{1}{8}$ pounds. Represent this situation with an expression involving signed numbers. What is the overall change in Mr. Mitchell's weight?

$-10 + 5\frac{1}{8}$

$= -10 + 5 + \frac{1}{8}$

$= (-10 + 5) + \frac{1}{8}$

$= (-5) + \frac{1}{8}$

$= -4\frac{7}{8}$

Mr. Mitchell's weight dropped by $4\frac{7}{8}$ pounds.

Exercise 5 (7 minutes)

MP.1

Students work with a partner to complete the following exercise. Students make sense of each step and come up with an alternate method of solving the problem.

After five minutes, class resumes as a whole group, and students volunteer verbal explanations and their own methods for solving the problem.

Exercise 5

Jamal is completing a math problem and represents the expression $-5\frac{5}{7} + 8 - 3\frac{2}{7}$ with a single rational number as shown in the steps below. Justify each of Jamal's steps. Then, show another way to solve the problem.

$$= -5\frac{5}{7} + 8 + \left(-3\frac{2}{7}\right)$$

$$= -5\frac{5}{7} + \left(-3\frac{2}{7}\right) + 8$$

$$= -5 + \left(-\frac{5}{7}\right) + (-3) + \left(-\frac{2}{7}\right) + 8$$

$$= -5 + \left(-\frac{5}{7}\right) + \left(-\frac{2}{7}\right) + (-3) + 8$$

$$= -5 + (-1) + (-3) + 8$$

$$= -6 + (-3) + 8$$

$$= (-9) + 8$$

$$= -1$$

Scaffolding:

Provide a list of the properties of operations as a reference sheet or as a bookmark for students.

Step 1:   *Subtracting a number is the same as adding its inverse.*

Step 2:   *Apply the commutative property of addition.*

Step 3:   *The opposite of a sum is the sum of its opposites.*

Step 4:   *Apply the commutative property of addition.*

Step 5:   *Apply the associative property of addition.*

$$\left(-\frac{5}{7}\right) + \left(-\frac{2}{7}\right) = \left(-\frac{7}{7}\right) = -1$$

Step 6: $-5 + (-1) = -6$

Step 7: $-6 + -3 = -9$

Step 8: $-9 + 8 = -1$

Answers will vary for other methods of reaching a single rational number. Students may choose to add $-5\frac{5}{7}$ and 8 together first, but a common mistake is to represent their sum as $3\frac{5}{7}$, rather than $2\frac{2}{7}$.

Closing (2 minutes)

- How can we rewrite the opposite of a sum?
  - *As the sum of its opposites*
- How is it helpful when finding the sums and differences of rational numbers to use the properties of operations?
  - *It allows us to regroup terms so that we can efficiently arrive at an answer. For instance, in an expression we may wish to first combine certain rational numbers that are in decimal form or those that are in fractional form. Or, we may wish to group together all the negative numbers if we are finding the sum of positive and negative numbers.*

Lesson Summary

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance,
$$-5\frac{2}{9} + 3.7 + 5\frac{2}{9} = \left(-5\frac{2}{9} + 5\frac{2}{9}\right) + 3.7 = 0 + 3.7 = 3.7.$$

- The opposite of a sum is the sum of its opposites as shown in the examples that follow:
$$-4\frac{4}{7} = -4 + \left(-\frac{4}{7}\right).$$
$$-(5 + 3) = -5 + (-3).$$

Exit Ticket (4 minutes)

Name _____   Date _____

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Exit Ticket

Mariah and Shane both started to work on a math problem and were comparing their work in math class. Are both of their representations correct? Explain, and finish the math problem correctly to arrive at the correct answer.

Math Problem

Jessica's friend lent her \$5. Later that day Jessica gave her friend back $1\frac{3}{4}$ dollars.

Which rational number represents the overall change to the amount of money Jessica's friend has?

Mariah started the problem as follows:

$$-5 - \left(-1\frac{3}{4}\right)$$

$$= -5 + 1 - \frac{3}{4}$$

Shane started the problem as follows:

$$-5 - \left(-1\frac{3}{4}\right)$$

$$= -5 + \left(1\frac{3}{4}\right)$$

$$= -5 + \left(1 + \frac{3}{4}\right)$$

_____

_____

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Exit Ticket Sample Solutions

Mariah and Shane both started to work on a math problem and were comparing their work in math class. Are both of their representations correct? Explain, and finish the math problem correctly to arrive at the correct answer.

Math Problem

Jessica's friend lent her $5. Later that day Jessica gave her friend back $1\frac{3}{4}$ dollars.

Which rational number represents the overall change to the amount of money Jessica's friend has?

Mariah started the problem as follows:

$$-5 - \left(-1\frac{3}{4}\right)$$

$$= -5 + 1 - \frac{3}{4}$$

Shane started the problem as follows:

$$-5 - \left(-1\frac{3}{4}\right)$$

$$= -5 + \left(1\frac{3}{4}\right)$$

$$= -5 + \left(1 + \frac{3}{4}\right)$$

Shane's method is correct. In Mariah's math work, she only dealt with part of the mixed number. The fractional part should have been positive too because the opposite of $-1\frac{3}{4}$ is $1\frac{3}{4}$, which contains both a positive 1 and a positive $\frac{3}{4}$. The correct work would be

$$-5 - \left(-1\frac{3}{4}\right) = -5 + \left(1\frac{3}{4}\right) = -5 + \left(1 + \frac{3}{4}\right) = (-5 + 1) + \frac{3}{4} = -4 + \frac{3}{4} = -3\frac{1}{4}.$$

The rational number would be $-3\frac{1}{4}$, which means Jessica's friend gave away $3\frac{1}{4}$ dollars, or $3.25.

Problem Set Sample Solutions

1. Represent each sum as a single rational number.

   a. $-14 + \left(-\frac{8}{9}\right)$

   $-14\frac{8}{9}$

   b. $7 + \frac{1}{9}$

   $7\frac{1}{9}$

   c. $-3 + \left(-\frac{1}{6}\right)$

   $-3\frac{1}{6}$

Rewrite each of the following to show that *the opposite of a sum is the sum of the opposites.* Problem 2 has been completed as an example.

2. $-(9 + 8) = -9 + (-8)$

   $-17 = -17$ *Answer provided in student materials.*

3. $-\left(\frac{1}{4}+6\right) = -\frac{1}{4} + (-6)$

$$-6\frac{1}{4} = -6\frac{1}{4}$$

4. $-(10 + (-6)) = -10 + 6$

$$-4 = -4$$

5. $-\left((-55) + \frac{1}{2}\right) = 55 + \left(-\frac{1}{2}\right)$

$$54\frac{1}{2} = 54\frac{1}{2}$$

Use your knowledge of rational numbers to answer the following questions.

6.   Meghan said the opposite of the sum of -12 and 4 is 8. Do you agree? Why or why not?

   Yes, I agree. The sum of -12 and 4 is -8, and the opposite of -8 is 8.

7.   Jolene lost her wallet at the mall. It had $\$10$ in it. When she got home, her brother felt sorry for her and gave her $\$5.75$. Represent this situation with an expression involving rational numbers. What is the overall change in the amount of money Jolene has?

   $-10 + 5.75 = -4.25$ *The overall change in the amount of money Jolene has is -4.25 dollars.*

8.   Isaiah is completing a math problem and is at the last step: $25 - 28\frac{1}{5}$. What is the answer? Show your work.

   $$25 - 28\frac{1}{5} = 25 + \left(-28 + \left(-\frac{1}{5}\right)\right) = (25 + -28) + \left(-\frac{1}{5}\right) = -3\frac{1}{5}$$

9.   A number added to its opposite equals zero. What do you suppose is true about *a sum added to its opposite*?

   Use the following examples to reach a conclusion. Express the answer to each example as a single rational number.

   A sum added to its opposite is zero.

   a. $(3 + 4) + (-3 + -4) = 7 + (-7) = 0$

   b. $(-8 + 1) + (8 + (-1)) = (-7) + 7 = 0$

   c. $\left(-\frac{1}{2} + \left(-\frac{1}{4}\right)\right) + \left(\frac{1}{2} + \frac{1}{4}\right) = \left(-\frac{3}{4}\right) + \frac{3}{4} = 0$

Number Correct: _____

Integer Addition—Round 1

Directions: Determine the sum of the integers, and write it in the column to the right.

1.	$8 + (-5)$		18.	$-38 + 25$	
2.	$10 + (-3)$		19.	$-19 + (-11)$	
3.	$2 + (-7)$		20.	$2 + (-7)$	
4.	$4 + (-11)$		21.	$-23 + (-23)$	
5.	$-3 + (-9)$		22.	$45 + (-32)$	
6.	$-12 + (-7)$		23.	$16 + (-24)$	
7.	$-13 + 5$		24.	$-28 + 13$	
8.	$-4 + 9$		25.	$-15 + 15$	
9.	$7 + (-7)$		26.	$12 + (-19)$	
10.	$-13 + 13$		27.	$-24 + (-32)$	
11.	$14 + (-20)$		28.	$-18 + (-18)$	
12.	$6 + (-4)$		29.	$14 + (-26)$	
13.	$10 + (-7)$		30.	$-7 + 8 + (-3)$	
14.	$-16 + 9$		31.	$2 + (-15) + 4$	
15.	$-10 + 34$		32.	$-8 + (-19) + (-11)$	
16.	$-20 + (-5)$		33.	$15 + (-12) + 7$	
17.	$-18 + 15$		34.	$-28 + 7 + (-7)$	

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Integer Addition—Round 1 [KEY]

Directions: Determine the sum of the integers, and write it in the column to the right.

1.	$8 + (-5)$	**3**	18.	$-38 + 25$	**−13**
2.	$10 + (-3)$	**7**	19.	$-19 + (-11)$	**−30**
3.	$2 + (-7)$	**−5**	20.	$2 + (-7)$	**−5**
4.	$4 + (-11)$	**−7**	21.	$-23 + (-23)$	**−46**
5.	$-3 + (-9)$	**−12**	22.	$45 + (-32)$	**13**
6.	$-12 + (-7)$	**−19**	23.	$16 + (-24)$	**−8**
7.	$-13 + 5$	**−8**	24.	$-28 + 13$	**−15**
8.	$-4 + 9$	**5**	25.	$-15 + 15$	**0**
9.	$7 + (-7)$	**0**	26.	$12 + (-19)$	**−7**
10.	$-13 + 13$	**0**	27.	$-24 + (-32)$	**−56**
11.	$14 + (-20)$	**−6**	28.	$-18 + (-18)$	**−36**
12.	$6 + (-4)$	**2**	29.	$14 + (-26)$	**−12**
13.	$10 + (-7)$	**3**	30.	$-7 + 8 + (-3)$	**−2**
14.	$-16 + 9$	**−7**	31.	$2 + (-15) + 4$	**−9**
15.	$-10 + 34$	**24**	32.	$-8 + (-19) + (-11)$	**−38**
16.	$-20 + (-5)$	**−25**	33.	$15 + (-12) + 7$	**10**
17.	$-18 + 15$	**−3**	34.	$-28 + 7 + (-7)$	**−28**

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Number Correct: _____

Improvement: _____

Integer Addition—Round 2

Directions: Determine the sum of the integers, and write it in the column to the right.

1.	$5 + (-12)$	
2.	$10 + (-6)$	
3.	$-9 + (-13)$	
4.	$-12 + 17$	
5.	$-15 + 15$	
6.	$16 + (-25)$	
7.	$-12 + (-8)$	
8.	$-25 + (-29)$	
9.	$28 + (-12)$	
10.	$-19 + (-19)$	
11.	$-17 + 20$	
12.	$8 + (-18)$	
13.	$13 + (-15)$	
14.	$-10 + (-16)$	
15.	$35 + (-35)$	
16.	$9 + (-14)$	
17.	$-34 + (-27)$	

18.	$23 + (-31)$	
19.	$-26 + (-19)$	
20.	$16 + (-37)$	
21.	$-21 + 14$	
22.	$33 + (-8)$	
23.	$-31 + (-13)$	
24.	$-16 + 16$	
25.	$30 + (-43)$	
26.	$-22 + (-18)$	
27.	$-43 + 27$	
28.	$38 + (-19)$	
29.	$-13 + (-13)$	
30.	$5 + (-8) + (-3)$	
31.	$6 + (-11) + 14$	
32.	$-17 + 5 + 19$	
33.	$-16 + (-4) + (-7)$	
34.	$8 + (-24) + 12$	

Integer Addition—Round 2 [KEY]

Directions: Determine the sum of the integers, and write it in the column to the right.

1.	$5 + (-12)$	**−7**	18.	$23 + (-31)$	**−8**
2.	$10 + (-6)$	**4**	19.	$-26 + (-19)$	**−45**
3.	$-9 + (-13)$	**−22**	20.	$16 + (-37)$	**−21**
4.	$-12 + 17$	**5**	21.	$-21 + 14$	**−7**
5.	$-15 + 15$	**0**	22.	$33 + (-8)$	**25**
6.	$16 + (-25)$	**−9**	23.	$-31 + (-13)$	**−44**
7.	$-12 + (-8)$	**−20**	24.	$-16 + 16$	**0**
8.	$-25 + (-29)$	**−54**	25.	$30 + (-43)$	**−13**
9.	$28 + (-12)$	**16**	26.	$-22 + (-18)$	**−40**
10.	$-19 + (-19)$	**−38**	27.	$-43 + 27$	**−16**
11.	$-17 + 20$	**3**	28.	$38 + (-19)$	**19**
12.	$8 + (-18)$	**−10**	29.	$-13 + (-13)$	**−26**
13.	$13 + (-15)$	**−2**	30.	$5 + (-8) + (-3)$	**−6**
14.	$-10 + (-16)$	**−26**	31.	$6 + (-11) + 14$	**9**
15.	$35 + (-35)$	**0**	32.	$-17 + 5 + 19$	**7**
16.	$9 + (-14)$	**−5**	33.	$-16 + (-4) + (-7)$	**−27**
17.	$-34 + (-27)$	**−61**	34.	$8 + (-24) + 12$	**−4**

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